Environmental Effects on Nonlocal Correlations

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Environmental interactions are ubiquitous in practical instances of any quantum information processing protocol. The interaction results in depletion of various quantum resources and even complete loss in numerous situations. Nonlocality, which is one particular quantum resource marking a significant departure of quantum mechanics from classical mechanics, meets the same fate. In the present work we study the decay in nonlocality to the extent of the output state admitting a local hidden state model. Using some fundamental quantum channels we also demonstrate the complete decay in the resources in the purview of the Bell–Clauser–Horne–Shimony–Holt inequality and a three-settings steering inequality. We also obtain bounds on the parameter of the depolarizing map for which it becomes steerability breaking pertaining to a general class of two qubit states.
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1 Introduction

Nonlocality [1] is one of the key features that sets quantum mechanics apart from classical mechanics. Quantum nonlocality is generally interpreted as the failure to describe quantum mechanical correlations arising due to spacelike separated local quantum measurements on subsystems of a composite system by local realist models. Bell inequalities [2] are used to reveal such incompatibility between quantum mechanics and local-realism. A state which satisfies a set of Bell inequalities cannot be guaranteed as local, as there may exist another set of Bell inequalities that it violates. On the other hand, quantum mechanical violation of any Bell–Clauser–Horne–Shimony–Holt (Bell–CHSH) inequality is a signature of quantum nonlocality [3]. However, the complete set of Bell–CHSH inequalities is the necessary and sufficient criterion for local-realism in the 2–2–2 experimental scenario (2 parties, 2 measurement settings per party, 2 outcomes per measurement setting). A state is termed as local only if the correlations arising by performing local quantum measurements on it admit a local hidden variable (LHV) model [4].

The pioneering study by Einstein, Podolsky and Rosen (EPR) [5] arguing for incompleteness of the quantum

mechanical description of *reality* motivated Schrödinger to propose the concept of *quantum steering* [6, 7]. EPR steering arises in the scenario where local quantum measurements on one part of a bipartite spatially separated system allow to prepare different ensembles on the other part. This scenario demonstrates EPR steering if these ensembles cannot be explained by a local hidden state (LHS) model [8, 9]. This kind of interpretation of steering has induced great interest in foundational research in recent times as evidenced by a wide range of studies [10–20]. Reid first proposed a criterion for testing EPR-steering in continuous-variable systems based on position-momentum uncertainty relation [21], which was experimentally tested by Ou *et al*. [22]. More recently, Cavalcanti *et al*. constructed experimental EPR-steering criteria based on the assumption of the existence of LHS model [23]. This general construction is applicable to discrete as well as continuous-variable observables and Reid's criterion appears as a special case of this general formulation.

More importantly, entanglement [24] is a necessary condition for the demonstration of Bell-nonlocality or steerability. The inequivalence of entanglement and nonlocality is exemplified by showing the existence of certain entangled states producing quantum mechanical correlations that admit a LHV model [25]. Bell-nonlocal states form a strict subset of steerable states which also form a strict subset of entangled states [8, 26].

Apart from being important candidates in foundational studies of quantum mechanics, quantum nonlocal, steerable and entangled states serve as resource in various quantum information processing tasks, for example, teleportation [27], randomness certification [28–30], cryptography [31, 32] and so on. Motivated by this fact a number of studies have been performed towards revealing *hidden* nonlocality from quantum states that failed to demonstrate nonlocality under the standard Bell scenario [1]. Local filtering operation is one such procedure [33, 34], which can be broadly classified into two categories: (i) performing single local measurement [34] and (ii) subjecting the state to suitable sequence of local measurements [33]. Similarly, the issue of revealing *hidden* quantum steerability by using local filters has also been studied [26, 35].

In practical scenarios, a state is subjected to ubiquitous environmental interaction and hence may lose its entanglement or nonlocal character partially or completely. Thus, it is of considerable interest to study the behaviour of entangled states as well as nonlocal states under local noise. The issue related to entanglement breaking channels which transform an entangled state into a separable one, has acquired a lot of significance as witnessed by a number of studies [36, 37]. On the other hand, studies on nonlocality breaking maps, which transform a

Bell–CHSH nonlocal state into a local one, have also probed the role of environment in destroying quantum resources [38]. Recently, a general framework for analyzing resource theories based on resource destroying maps has been proposed [39]. In this context, one may also consider the case of incompatibility breaking maps [40], as incompatibility of quantum measurements is an important resource in quantum information. The study of such maps is intriguing as steerability and incompatibility have a one to one correspondence [41].

A separable state can be transformed into an entangled one when subjected to a global unitary action acting on the composite system. However, it has been shown that there are separable states, dubbed as absolutely separable states, which cannot be transformed into an entangled one under any global unitary interaction [42–45]. Recently, the effect of global unitary interactions on the nonlocality of a state has been probed, with the focus on the Bell–CHSH inequality for two qubit systems. A state initially satisfying the Bell–CHSH inequality can violate it after a global unitary interaction. On the other hand, it has been demonstrated that there are states which preserve their Bell–CHSH local character under arbitrary global unitary action. These states are termed as absolutely Bell–CHSH local states [46, 47]. The question of transforming a separable, but not absolutely separable, state into an absolutely separable one under environmental interactions is important in practical situations and has been investigated recently [48].

In the context of absolutely Bell–CHSH local states, the issue of transforming an absolutely Bell–CHSH local state into a nonlocal one has also been presented in a recent study [49]. In a similar spirit, the effect of global unitary interactions on states demonstrating EPR steering has also been studied [50] in the context of steering inequalities derived by Cavalcanti *et al*. [23]. In particular, the issue of non-violation of the steering inequality with three measurement settings per party [23] by any two qubit system under arbitrary global unitary action has been studied in details. For our convenience, we will denote the states which preserve their non-violation of the steering inequality with three measurement settings per party derived by Cavalcanti *et al*. [23] under arbitrary global unitary action as *absolutely three-settings unsteerable states*. Studies have shown that any pure state cannot be absolutely Bell–CHSH local, or absolutely three-settings unsteerable, or absolutely separable. However, one must remember that environmental interactions bound us to work with mixed states where such phenomena are practically very possible. Therefore, one can also understand the significance of such confrontations even through the lens of quantum information processing protocols.

A natural question arising in this context is what are the possible instances that one should avoid while working with nonlocal resources under the influence of environment. In the present work we provide such illustrations with commonly used quantum channels. Our study is done in the purview of Bell–CHSH inequality and the three-settings steering inequality mentioned in the text. The resultant states do not violate the inequalities even after consuming the power of global unitary operations. This is where the present study departs from previous works as it probes the feature that nonlocality may be non-retrievable even with global unitary operations.

On the foundational significance of our work, we have been able to generate entangled states which admit LHS models from initially nonlocal state. Werner, in his seminal paper [25] had proved the existence of entangled states having LHV. That is, entangled states exist which cannot give rise to nonlocality under arbitrary (non-sequential) measurements. There is a recent spurt in research in constructing entangled states having LHV or LHS model [51, 52] using semi-definite programming. To put matters in context, let Alice and Bob share a quantum state. Alice performs a measurement $x \in \{x_0, x_1, x_2, ..., x_n\}$ and obtains an outcome $a \in \{0, 1, 2, ..., d_A\}$ and Bob performs a measurement $y \in \{y_0, y_1, y_2, ..., y_m\}$ and obtains an outcome $b \in \{0, 1, 2, ..., d_B\}$. If they repeat these trials many times, they generate a correlation $p(ab|xy)$. The joint correlation is said to have LHV model if and only if $p(ab|xy) = \sum_\lambda \rho(\lambda)p(a|x, \lambda)p(b|y, \lambda)$, where $p(a|x, \lambda)$ and $p(b|y, \lambda)$ are arbitrary distributions conditioned on LHV λ. On the other hand, the joint correlation is said to have LHS model if and only if $p(ab|xy) = \sum_\lambda \rho(\lambda)p(a|x, \lambda)p(b|y, \sigma_\lambda)$, where $p(a|x, \lambda)$ is arbitrary distribution conditioned on LHV λ, $p(b|y, \sigma_\lambda)$ is the quantum probability of obtaining the outcome b when the measurement y is performed on the local hidden state σ_λ (which is a quantum state). In our present work we have been able to generate an entangled state admitting LHS model due to environmental influence.

Pure entangled states always violate a suitably chosen Bell's inequality. However, purity is fragile under environmental influences and thus, mixed states are more common in experimental scenarios. Mixed entangled states can have a LHV model, the most celebrated example being of the Werner state in two qubits. Therefore, our present work deals with mixed entangled states.

We have studied the context of steerability breaking maps. Particularly we have obtained bounds on the parameter of the depolarizing map such that it becomes steerability breaking pertaining to a general class of two qubit states. This is particularly significant because steerability is a weaker form of nonlocality as compared to Bell nonlocality.

The paper has been arranged in the following way. Starting from some preliminary discussions in Section 2 we have illustrated the scenarios in Section 3. In Section 4, we have studied the transformation of Bell–CHSH nonlocal state to Bell–CHSH local, absolutely Bell–CHSH local and absolutely three-settings unsteerable states under environmental interactions, followed by one of the highlights of our analysis, *viz.*, generation of entangled states admitting LHS model from initially nonlocal state using two different quantum channels in Section 5. In Section 6, we discuss on the analysis of steerability breaking channels in the context of the depolarizing map which is another interesting attribute of our approach. Finally, we conclude along with a discussion in Section 7.

2 Preliminaries

Let us start with some preliminary ideas required for the present study.

2.1 Bell–CHSH locality

A bipartite state is said to be Bell–CHSH local if and only if (iff) the correlations obtained by performing local quantum measurements on the two subsystems of the composite state (where the local measurements performed on one subsystem are spacelike separated from that on another subsystem) do not violate the Bell–CHSH inequality. The necessary and sufficient criteria for quantum mechanical violation of the CHSH inequality by arbitrary bipartite qubit states has been established in [53].

An arbitrary two qubit state can be expressed in terms of the Hilbert–Schmidt basis as

$$\rho = \frac{1}{4}(\mathbb{I} \otimes \mathbb{I} + \vec{r}.\vec{\sigma} \otimes \mathbb{I} + \mathbb{I} \otimes \vec{s}.\vec{\sigma} + \sum_{i,j=1}^{3} t_{ij}\sigma_i \otimes \sigma_j). \quad (1)$$

Here $\mathbb{I}$ is the identity operator acting on $\mathbb{C}^2$; σ_is are the three Pauli matrices; $\vec{r}, \vec{s}$ are vectors in $\mathbb{R}^3$ with norm less than or equal to unity; $\vec{r}.\vec{\sigma} = \sum_{i=1}^{3} r_i\sigma_i$ and $\vec{s}.\vec{\sigma} = \sum_{i=1}^{3} s_i\sigma_i$. The condition $\text{Tr}(\rho^2) \leq 1$ implies

$$\sum_{i=1}^{3} \left(r_i^2 + s_i^2\right) + \sum_{i,j=1}^{3} t_{ij}^2 \leq 3, \quad (2)$$

where the equality is achieved for the pure states. In addition, for being a valid density matrix, ρ has to be positive semidefinite.

Let us consider the matrix $V = T^\dagger T$, where T is the correlation matrix of the state (1) with matrix elements $t_{ij} = \text{Tr}(\rho\sigma_i \otimes \sigma_j)$. Denote with u_1, u_2 the two greatest eigenvalues of V. Let us consider the quantity given by

$$M(\rho) = u_1 + u_2. \quad (3)$$

The state given by Eq. (1) violates the Bell–CHSH inequality iff $M(\rho) > 1$. Hence, the state (1) is Bell–CHSH local iff $M(\rho) \leq 1$.

2.2 Absolute Bell–CHSH locality

The concept of absolutely Bell–CHSH local states has recently been introduced in [46]. A Bell–CHSH local quantum state is said to be absolutely Bell–CHSH local if the state remains Bell–CHSH local under the action of any global unitary operation. If a_1, a_2, a_3 are the three largest eigenvalues of the given two qubit state ρ taken in descending order, then the state ρ is absolutely Bell–CHSH local iff [47]

$$A(\rho) = (2a_1 + 2a_2 - 1)^2 + (2a_1 + 2a_3 - 1)^2 \leq 1. \quad (4)$$

2.3 Absolute three-settings unsteerability

Cavalcanti *et al*. have provided a series of steering inequalities to certify whether a bipartite state is steerable when each of the two parties are allowed to perform n measurements on his or her part [23]. In particular for $n = 3$, the inequality is given by,

$$F^3 = \frac{1}{\sqrt{3}} \left| \sum_{i=1}^{3} \langle A_i \otimes B_i \rangle \right| \leq 1, \quad (5)$$

where, $A_i = \hat{u}_i . \vec{\sigma}$, $B_i = \hat{v}_i . \vec{\sigma}$, $\vec{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$ is a vector composed of Pauli matrices, $\hat{u}_i \in \mathbb{R}^3$ are unit vectors, $\hat{v}_i \in \mathbb{R}^3$ are orthonormal vectors. $\langle A_i \otimes B_i \rangle = \text{Tr}(\rho A_i \otimes B_i)$ with $\rho \in \mathcal{B}(\mathcal{H}_A \otimes \mathcal{H}_B)$ is the bipartite quantum system shared between the two parties.

The states, which preserve their non-violation of the steering inequality (5) under arbitrary global unitary action, are called *absolutely three-settings unsteerable states*. A given two qubit state ρ is absolutely three-settings unsteerable iff [50]

$$B(\rho) = 3\text{Tr}(\rho^2) - 2(x_1 x_2 + x_1 x_3 + x_1 x_4$$
$$+ x_2 x_3 + x_2 x_4 + x_3 x_4) \leq 1, \quad (6)$$

where x_i are the eigenvalues of the two qubit state ρ.

2.4 Quantum channels

In practical scenarios it is very hard to isolate a qubit from its environment. Environmental interactions can be represented by different quantum channels. Quantum channels are completely positive trace preserving (CPTP) maps acting on the space of density matrices [54]. Every quantum channel admits the operator sum representation.

Let ε be a quantum channel, then its action on the state $\tilde{\rho}$ can be expressed as

$$\varepsilon(\tilde{\rho}) = \sum_i K_i \tilde{\rho} K_i^\dagger, \quad (7)$$

where K_i's are Kraus operators for the corresponding channel with $\sum_i K_i^\dagger K_i = \mathbb{I}$ ($\mathbb{I}$ is the identity operator). In the present study we restrict ourselves to four quantum channels, viz. 1) phase-flip channel , 2) bit-flip channel, 3) depolarizing channel, and 4) phase damping channel.

2.4.1 Phase-flip channel

The action of the phase-flip channel on the state $\tilde{\rho}$ is given by Eq. (7) with the following Kraus operators [54]

$$K_0 = \sqrt{1 - p}\mathbb{I},$$
$$K_1 = \sqrt{p}\sigma_z,$$

where p is the channel strength with $0 \leq p \leq 1$.

2.4.2 Bit-flip channel

The action of the bit-flip channel on the state $\tilde{\rho}$ is given by Eq. (7) with the following Kraus operators [54]

$$K_0 = \sqrt{1 - p}\mathbb{I},$$
$$K_1 = \sqrt{p}\sigma_x$$

where p is the channel strength with $0 \leq p \leq 1$.

2.4.3 Depolarizing channel

The action of the depolarizing channel on the state $\tilde{\rho}$ is given by Eq. (7) with the following Kraus operators [54]

$$K_0 = \sqrt{1 - p}\mathbb{I},$$
$$K_1 = \sqrt{\frac{p}{3}}\sigma_x,$$
$$K_2 = \sqrt{\frac{p}{3}}\sigma_y,$$
$$K_3 = \sqrt{\frac{p}{3}}\sigma_z,$$

where p is the channel strength with $0 \leq p \leq 1$.

2.4.4 Phase damping channel

The action of the phase damping channel on the state $\tilde{\rho}$ is given by Eq. (7) with the following Kraus operators [54]

$$K_0 = \begin{bmatrix} 1 & 0 \\ 0 & \sqrt{1-p} \end{bmatrix},$$
$$K_1 = \begin{bmatrix} 0 & 0 \\ 0 & \sqrt{p} \end{bmatrix}$$

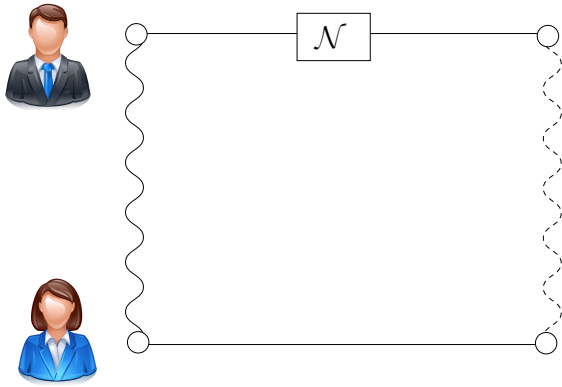Here p is the channel strength with $0 \leq p \leq 1$.

Figure 1: *Single interaction scenario where N stands for local channel acting on Bob's side.*
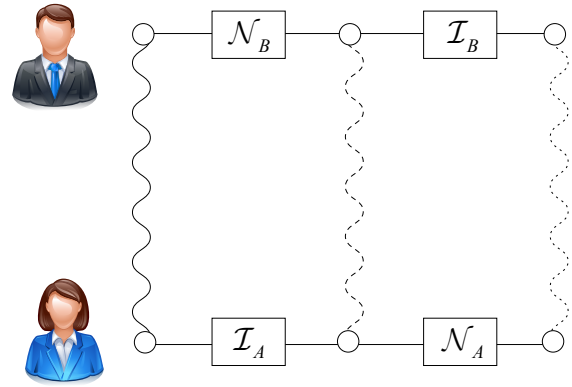

Figure 2: *Sequential interaction where $\mathcal{N}_B$ and $\mathcal{N}_A$ stand for local channels acting on Bob's and Alice's side respectively. $\mathcal{I}_B$ and $\mathcal{I}_A$ represent identity operations on respective sides.*

3 The Scenarios

In order to study the effect of environmental interactions on nonlocal states, we have considered two scenarios as follows.

3.1 Single interaction

Consider that Alice and Bob (the set of local measurements performed by Alice is spacelike separated from that of Bob) share a nonlocal two qubit mixed state. Bob's qubit interacts with the environment simulated by a quantum channel of strength p. They finally obtain a family of mixed states which are dependent on p. We calculate the range of the channel strengths for which the state becomes Bell–CHSH local, absolutely Bell–CHSH local, absolutely three-settings unsteerable. Figure 1 depicts the scenario.

3.2 Double Interaction

In this case the situation differs from the above in the sense that now both the qubits interact with environment. The quantum channel is of the same strength p. In this case too we obtain the spread of the channel strength pertaining to the same manifestations as above. The scenario is represented by Figure 2.

Next, we present the first significant result of our work.

4 From nonlocal state to Bell–CHSH local state, absolutely Bell–CHSH local state and absolutely three-settings unsteerable state

Let us consider that the following two parameter family of mixed states is shared among Alice and Bob where the local measurements performed by Alice are spacelike separated from that of Bob

$$
\rho_i(\lambda, \theta) = \begin{bmatrix} \frac{1-\lambda}{2} & 0 & 0 & 0 \\ 0 & \lambda \sin^2\theta & \frac{\lambda}{2}\sin 2\theta & 0 \\ 0 & \frac{\lambda}{2}\sin 2\theta & \lambda\cos^2\theta & 0 \\ 0 & 0 & 0 & \frac{1-\lambda}{2} \end{bmatrix}. \tag{8}
$$

Figure 3 depicts the nonlocal region for the family of mixed states given by Eq. (8). Let us choose two initial states from the above two parameter family of states, one with $\lambda = 0.95$, $\theta = 0.6$ and another with $\lambda = 0.80$ and $\theta = 0.6$, such that the two initial states are nonlocal. Let us consider that the initial states are subjected to the aforementioned single and double interactions of different quantum channels with channel strength p. Let R_1, R_2 and R_3 denote the ranges of p for which the states obtained are Bell–CHSH local, absolutely Bell–CHSH local, and absolutely three-settings unsteerable, respectively.

In what follows below we have tabulated the various manifestations under different channel parameters for the single and double interactions respectively.

Tables 1 and 2 show the effect of single and sequential interaction of different channels with respect to two different initial states.

It is significant to note that for *bit-flip* and *phase-flip* channels, along with the lower bound there is also an upper bound on the noise parameter (p), between which they
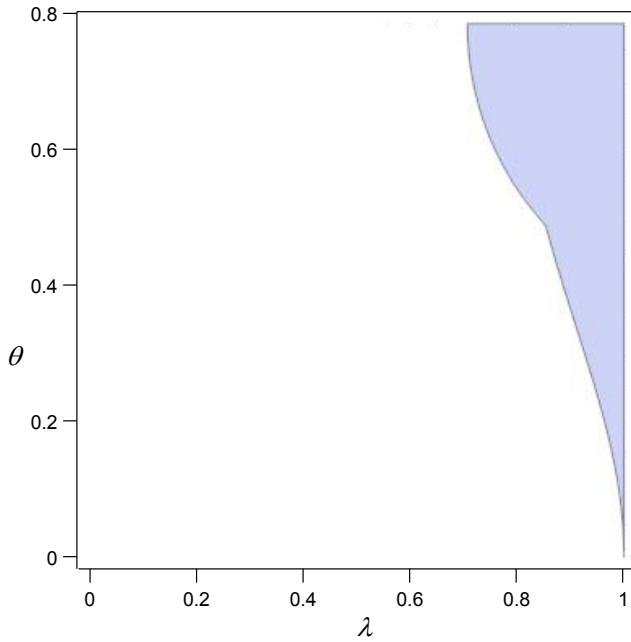
Figure 3: *The horizontal axis represents λ and the vertical axis represents θ. The shaded region depicts the nonlocal region of the state (8).*

relabelling of the initial quantum state. As a result, for the value of noise parameter beyond an upper-bound, the resource destroying power of the map is decreased in case of the above mentioned channels. It is also worthwhile to note that barring the first row and one range in the second row the ranges in Table 1 are subsets of their counterpart in Table 2. This agrees with intuition as Table 2 depicts the range pertaining to double interaction.

5 Generating entangled states admitting LHS model from initially nonlocal state

There has been considerable work in recent times on the construction of states admitting LHV and LHS models using techniques from semidefinite programming [51, 52]. In what follows below, we lay down two scenarios which generate entangled states admitting LHS model from initially nonlocal state.

5.1 The scenario using phase damping channel

act as resource destroying maps. This counter-intuitive feature has a deep connection with the structure of the state (8). Single interaction of the *phase flip* noise makes $(\lambda, \theta) \rightarrow (\lambda, -\theta)$, whereas the *bit flip* noise makes a block

Let us consider the following two parameter family of bipartite two-qubit mixed quantum states initially shared between Alice and Bob (the local measurements performed

Table 1: *Single interaction*

Channels	Initial state parameters	R_1	R_2	R_3
Phase flip	$\lambda = 0.95, \theta = 0.6$	[0.1492, 0.8508]	[0.2252, 0.7747]	–
Phase flip	$\lambda = 0.8, \theta = 0.6$	[0.0258, 0.9742]	[0.0675,0.9325]	[0.1743, 0.8257]
Bit flip	$\lambda = 0.95, \theta = 0.6$	[0.2445,0.7342]	[0.3532,0.6443]	[0.3807,0.6193]
Bit flip	$\lambda = 0.8, \theta = 0.6$	[0.0531,0.9468]	[0.1250, 0.8750]	[0.2, 0.8]
Depolarizing	$\lambda = 0.95, \theta = 0.6$	[0.0685, 1]	[0.1928,1]	[0.2893,1]
Depolarizing	$\lambda = 0.8, \theta = 0.6$	[0.0692,1]	[0.1928, 1]	[0.2893,1]
Phase damping	$\lambda = 0.95, \theta = 0.6$	[0.3723,1]	[0.7846, 1]	–
Phase damping	$\lambda = 0.8, \theta = 0.6$	[0.0516,1]	[0.1350, 1]	[0.3485, 1]

Table 2: *Double interaction*

Channels	Initial state parameters	R_1	R_2	R_3
Phase flip	$\lambda = 0.95, \theta = 0.6$	[0.1492, 0.8508]	[0.2252, 0.7747]	–
Phase flip	$\lambda = 0.8, \theta = 0.6$	[0.0131, 0.9869]	[0.3050,0.9650]	[0.0964, 0.9036]
Bit flip	$\lambda = 0.95, \theta = 0.6$	[0.1378,0.8622]	[0.1856,0.8144]	[0.2256,0.7744]
Bit flip	$\lambda = 0.8, \theta = 0.6$	[0.0273,0.9727]	[0.0654, 0.9345]	[0.1093, 0.8907]
Depolarizing	$\lambda = 0.95, \theta = 0.6$	[0.0354, 1]	[0.0727,1]	[0.1560,1]
Depolarizing	$\lambda = 0.8, \theta = 0.6$	[0.0196,1]	[0.0481, 1]	[0.0922,1]
Phase damping	$\lambda = 0.95, \theta = 0.6$	[0.2077,1]	[0.5359, 1]	–
Phase damping	$\lambda = 0.8, \theta = 0.6$	[0.0261,1]	[0.0699, 1]	[0.1928, 1]

by Alice are spacelike separated from that of Bob)

$$\rho_i(q, s) = q\left(s|\phi^+\rangle\langle\phi^+| + (1 - s)\frac{1}{4}\mathbb{I}\right)$$
$$+ (1 - q)\frac{1}{2}\left(|00\rangle\langle00| + |11\rangle\langle11|\right) \quad (9)$$

where $|0\rangle$ and $|1\rangle$ are the eigenstates of the operator σ_z with eigenvalues $+1$ and -1 respectively,

$$|\phi^+\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right),$$

$\mathbb{I}$ is the identity operator, $0 \le q \le 1$ and $0 \le s \le 1$.

Now, we use the phase damping channel. Let us choose $q = 0.96$, $s = 0.74$ such that the initial state given by Eq. (9) is nonlocal. In case of the phase damping channel with channel strength p, if this state undergoes single interaction as described earlier, then for $p = p_1 = 0.65$, the state becomes

$$\rho_f = \frac{1}{2}\sigma + \frac{1}{4}\left(|00\rangle\langle00| + |11\rangle\langle11|\right), \quad (10)$$

where σ is the two qubit isotropic state given by

$$\sigma = \frac{1}{2}\left(|\phi^+\rangle\langle\phi^+| + \frac{1}{4}\mathbb{I}\right).$$

It has been shown that the correlation produced by the state ρ_f given by Eq. (10) has a local hidden variable-local hidden state (LHV-LHS) model [55]. Hence, the correlation produced by the state ρ_f has a LHV model, since the states having a LHV-LHS model form a subset of the states having a LHV model. It can easily be checked that the state ρ_f is absolutely Bell–CHSH local according to the condition given by (4). The state ρ_f is absolutely three-settings unsteerable also according to the condition given by (6).

Again, in case of the phase damping channel with channel strength p, if the state $\rho_i(q, s)$ given by Eq. (9) (with $q = 0.96$, $s = 0.74$) undergoes sequential interaction as described earlier, then for $p = p_2 = 0.41$ the state becomes ρ_f given by Eq. (10).

Hence, here we have presented the transformation of a nonlocal state into an absolutely Bell–CHSH local state (as well as absolutely three-settings unsteerable state) with an LHV model as well as LHS model under single and double interaction of the phase damping channel. It is clear that $p_2 < p_1$, which implies that the state $\rho_i(q, s)$ (with $q = 0.96$, $s = 0.74$) can be transformed into an absolutely Bell–CHSH local state (as well as an absolutely three-settings unsteerable state) having a LHV model as well as LHS model under double interaction of the phase damping channel with smaller channel strength compared to that under single interaction.

5.2 The scenario using depolarizing channel

In this case let us choose $q = 0.34$, $s = 0.97$ such that the initial state given by Eq. (9) is nonlocal. In case of the depolarizing channel with channel strength p, if this state undergoes single interaction as described earlier, then for $p = p_1 = 0.18$ the state becomes ρ_f given by Eq. (10). If the state $\rho_i(q, s)$ given by Eq. (9) (with $q = 0.34$, $s = 0.97$) undergoes sequential interaction of the depolarizing channel as described earlier, then for $p = p_2 = 0.10$ the state becomes ρ_f given by Eq. (10).

Hence, here we have presented the transformation of a nonlocal state into an absolutely Bell–CHSH local state (as well as absolutely three-settings unsteerable state) having a LHV model as well as LHS model under single and double interaction of the depolarizing channel. Note that since, $p_2 < p_1$, the state $\rho_i(q, s)$ (with $q = 0.34$, $s = 0.97$) can be transformed into an absolutely Bell–CHSH local state (as well as an absolutely three-settings unsteerable state) having an LHV model as well as LHS model under double interaction of the depolarizing channel with smaller channel strength compared to that under single interaction.

6 Steerability breaking channels and depolarizing map

In [55], Bhattacharya *et al.* gave a sufficient criterion for unsteerability pertaining to a state in two qubits. They considered the following state in two qubits

$$\chi = \frac{1}{4}\left(\mathbb{I} \otimes \mathbb{I} + \vec{a}.\vec{\sigma} \otimes \mathbb{I} + \sum_i t_{ii}\sigma_i \otimes \sigma_i\right) \quad (11)$$

where $\mathbb{I}$ is the identity operator acting on $\mathbb{C}^2$, σ_is are the three Pauli matrices, $i \in \{1, 2, 3\}$, $\vec{r}, \vec{a}$ is vector in $\mathbb{R}^3$ with norm less than or equal to unity, $\vec{a}.\vec{\sigma} = \sum_{i=1}^{3} a_i\sigma_i$ and $t_{ii} = \text{Tr}[\chi(\sigma_i \otimes \sigma_i)]$. The state in Eq. (11) is unsteerable if $\max_{\hat{x}}[(\vec{a}.\hat{x})^2 + 2\|T\hat{x}\|] \le 1$, where $\|\cdot\|$ is the Euclidean vector norm, $T = [t_{ij}]$ and $\hat{x}$ is a unit vector.

Since $\max_{\hat{x}}(\vec{a}.\hat{x})^2 = |\vec{a}|^2$ and $\max_{\hat{x}}\|T\hat{x}\| = \sqrt{\lambda_{\text{max}}}$, where λ_{max} is the largest eigenvalue of $T^\dagger T$, we obtain the condition as

$$|\vec{a}|^2 + 2\sqrt{\lambda_{\text{max}}} \le 1. \quad (12)$$

The depolarizing map is an important completely positive map in quantum information processing. It has been studied in the context of incompatibility breaking channels [40]. Its action on a single qubit system A is given by

$$\Gamma_\epsilon(A) = \epsilon A + (1 - \epsilon)\frac{1}{2}\text{Tr}[A]\mathbb{I} \quad (13)$$

where ϵ is the channel parameter. Now, consider the state given in Eq. (11). Then due to the action of the depolarizing map on the state we obtain,

$$(\Gamma_\epsilon \otimes \mathbb{I})\chi = \frac{1}{4}\left(\mathbb{I} \otimes \mathbb{I} + \epsilon(\vec{a}.\vec{\sigma}) + \sum_i \epsilon t_{ii}\sigma_i \otimes \sigma_i \right). \quad (14)$$

On using the unsteerability criterion, the resultant state is unsteerable if

$$|\epsilon\vec{a}|^2 + 2\epsilon \sqrt{\max(t_{11}^2, t_{22}^2, t_{33}^2)} \le 1. \quad (15)$$

This provides a bound on the parameter ϵ for which the map becomes steerability breaking. One may also consider the map to be incompatibility breaking [40], as steerability and incompatibility of measurements have a one to one correspondence [41]. However, one must note that since the unsteerability criterion is only sufficient, the bounds are not tight.

7 Conclusions

We have discussed the possible detrimental effects of the environment on nonlocal resources because in the context of any quantum information protocol one cannot ignore environmental influences. We have documented various scenarios where nonlocality is lost, sometimes to the extent that it cannot be retrieved even with the strong resource of global unitary operations. We have illustrated various channel actions which destroy certain correlations. From the perspective of experiments, one should avoid those maps to preserve various nonlocal correlations. It is in this line of thought, that our work assumes significance. Precisely we show that there are instances where a quantum state loses its ability to violate the Bell–CHSH inequality or the three-settings steering inequality. This contributes to the notion of nonlocality breaking channels and also focusses on steerability breaking maps.

We also attempted to further the understanding of absolutely local maps, which render the resultant state useless regarding nonlocality. This is akin to the work already done in entanglement theory where absolute separability maps were considered [48]. In the tables presented, as expected intuitively, we observe that in most of the cases the parameter ranges obtained for a single interaction are subsets of that obtained for double interaction. We would like to reiterate that our work concerns mixed entangled states as pure states always violate some suitably chosen Bell's inequality.

We have laid down a theoretical proposal to generate entangled states admitting LHS model. As such entangled states occupy an intriguing existence in the theory of non-locality and foundations of quantum theory, one might

consider the experimental demonstration of the result. We have obtained bounds on the parameter of the depolarizing map for which it becomes steerability breaking (or one may also term them as incompatibility breaking). This does not depend on any particular inequality. This work also leads to certain open questions and possibilities for subsequent studies. For example, one may probe other channel parameters for which they become steerability breaking. An extension of the work in higher dimensions and multipartite systems also deserves attention.

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