Can Decoherence Solve the Measurement Problem?

Mani L. Bhaumik

Department of Physics and Astronomy, University of California, Los Angeles, USA. E-mail: bhaumik@physics.ucla.edu

Editors: ***Zvi Bern & Danko Georgiev***

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The quantum decoherence program has become more attractive in providing an acceptable solution for the long-standing quantum measurement problem. Decoherence by quantum entanglement happens very quickly to entangle the quantum system with the environment including the detector. But in the final stage of measurement, acquiring the unentangled pointer states poses some problems. Recent experimental observations of the effect of the ubiquitous quantum vacuum fluctuations in destroying quantum entanglement appears to provide a solution.

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1 Introduction

Shortly after the formulation of quantum mechanics nearly a century ago, a rather strange aspect of the theory became apparent. It is variously known as the *quantum measurement problem*, *wave function collapse* or *wave function reduction*. This is because the non-relativistic quantum mechanics appears to consist of two essentially different processes. After a unitary evolution of the wave function of a microscopic quantum state following the linear Schrödinger equation, one must resort to a sudden non-unitary stochastic collapse of the wave function to obtain its classical measurement outcome.

The genesis of the problem started with the quantum pioneers headed by Niels Bohr. They insisted that we only ever observe any physical phenomena at the macroscopic level. We never directly deal with the quantum objects of the microscopic realm and therefore need not worry about them or their physical reality. Accordingly, they argued that both the observer and the measurement apparatus must be kept outside the system to which quantum mechanics is applied. This is known as the Copenhagen interpretation, which simply pronounced the issue of microscopic quantum states is out of bounds, stating that physicists just had to accept a fundamental distinction between the quantum and the classical domains. Without being disrespectful to the esteemed founding fathers of quantum mechanics, we may inquire how can such thoughts be ever consistent with a scientific outlook?

Nonetheless, this was epitomized by the mathematical mastermind John Von Neumann in his classic axiomatic formulation of non-relativistic quantum mechanics using the linear Hilbert vector space [1, 2]. Even after many decades, his skillful formulation is still taught in almost all advanced quantum mechanics classes despite the two obvious incongruities arising from the Copenhagen interpretation. These comprise of the essential *ad hoc* role of consciousness and the postulated assumption of an abrupt collapse of the wave function. Quantum mechanics itself does not predict the collapse, which must be manually added to the calculations. Einstein famously likened it to God playing dice to decide what becomes "real" – what we actually observe in our classical world. However, despite the quantum pioneers' assertions, enormous efforts by the physics community have been made leading to many alternate postulates to explain away the irrational

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proposal of the inventors. Significant progress has been made by these efforts but without leading to any consensus, although some substantial fissures have ensued in the original interpretation.

2 The collapse postulate

John von Neumann, way back in his formulation [1, 2], postulated his non-unitary “process 1”, to emphasize the role of *consciousness* for the collapse of the wave function in the measurement process. It was concluded by von Neumann and most of the physicists of the time that there is no physical reason for the collapse in measurement transition. Thus evolved the rather instinctive resort to the “consciousness of an external observer,” which appears to be fading in time. Even the stalwarts like Eugene Wigner fell for it [3] but eventually repudiated it later [4].

It is quite remarkable to note that so much effort by so many eminent scientists in the early years of quantum mechanics was devoted to the role of consciousness in quantum measurement. But this directly contradicts the obvious fact that in the early years of the universe, the conditions were not suitable for appearance of any manifest conscious agents. Yet the universe developed to a mature state obeying quantum rules long before the possibility of emergence of conscious beings. This has been characterized by John Bell in jest

> Was the wavefunction of the world waiting to jump for thousands of millions of years until a single-celled living creature appeared? Or did it have to wait a little longer, for some better qualified system . . . with a PhD? [5, p. 34]

To be fair, the conditions of the early universe were not known to the pioneers of quantum physics. It would be reasonable to speculate that very likely they would not have put such essential emphasis on consciousness if they knew the early universe conspicuously ascertains that consciousness is not essential for the workings of quantum rules.

The other enigmatic postulate that von Neumann institutionalized is the collapse of the wave function initially alluded by Werner Heisenberg. By then, it was well established that a quantum state in the macroscopic domain is usually a superposition of two or more states. But in measurement using classical devices, one observes only a single state and no superposition. Von Neumann conjectured that in the measurement process, the quantum states would collapse to one of the superposed states following his improvised projection postulate. Again, in John Bell’s words

> If the theory is to apply to anything but highly idealised laboratory operations, are we not obliged to admit that more or less ‘measurement-like’ processes are going on more or less all the time, more or less everywhere? [5, p. 34]

Gerhart Lüders rejected [6] von Neumann’s collapse postulate (except for degenerate states). Its confirmation came in a recent experiment performed by Pokorny *et al*. [7] who called it the ideal measurement process. In this well-planned experiment, the authors created a microscopic superposition of three quantum states. They were able to measure just one of the superposed states without collapsing the entire wave function also observing that the collapse happened over time and not instantaneously. Serge Haroche and his group [8] also demonstrated that reduction of the wave function happens gradually.

An example from the cosmic history is worth examining in this regard. The universe about 380 000 years after the big bang consisted primarily of hydrogen ions (protons) and electrons, along with neutral helium atoms. An electron would naturally be attracted to the proton, starting to emit electromagnetic radiation due to its motion. But a much more rapid process would take place when the electron, while aligned in the direction of the proton, spontaneously emits a virtual photon with an amount of energy that exactly matches the potential energy of the electron in an orbital of the hydrogen atom. In this process, the wave function of the electron can directly wind up as the wave function of a specific orbital of the hydrogen atom without having to undergo a typical collapse to any particular point. Such episodes would reveal that the wave function does not necessarily always need to go through a traditional collapse for detection.

But the mystery of the occurrence of the quantum to classical transition continues to persist. Consequently, substantial attempts have been made to find an acceptable solution by modifying the Schrödinger equation but without any success so far. Despite its outstanding success, some experts like Vittorio Gorini, Andrzej Kossakowski, George Sudarshan [9] and Göran Lindblad [10] have attempted to modify the Schrödinger equation to solve the measurement problem. Steven Weinberg using the Lindblad equation pointed out [11–13] using data from atomic clocks that any proposed modification would need to produce an accuracy of at least one part in 10^{17} in the difference between the energy states employed in the clock. The accuracy of the atomic clocks continues to improve requiring possibly even better improvement of the modification of the theory. So far, such approaches do not seem to be fruitful.

An attractive scheme generally known as the Ghirardi–Rimini–Weber theory [14, 15] has been studied extensively over the last four decades by arbitrarily attaching a Gaussian function to the Schrödinger equation. The modification acts as a Markovian process that has negligible effect during the unitary evolution but becomes active afterwards during the final measurement when a very large number of particles become available due to some unspecified diffusion process. The efficacy of the Ghirardi–Rimini–Weber modified Schrödinger equation remains to be demonstrated. It may be prudent to consider Steven Weinberg's contention

> Unfortunately, these ideas about modification of quantum mechanics are not only speculative but also vague, and we have no idea how big we should expect the corrections to quantum mechanics to be. [11, pp. 139–140]

Roger Penrose has proposed [16, 17] a novel scheme of a gravitational process to bring about the reduction of the wave function but without any successful experimental demonstration yet. The most fruitful approach now seems to be the one based on quantum decoherence. Furthermore, the decoherence time being relatively short [18], also seem to rule out the Ghirardi–Rimini–Weber modification and the gravitational reduction proposal since both will take some time to be built up for their effectiveness.

3 Further progress

It is rather amazing that not until about half a century after the advent of quantum mechanics, Heinz-Dieter Zeh was the first to emphasize that the microscopic quantum state wave function evolves unitarily obeying the Schrödinger equation in isolation from the environment [19]. However, for measurement, the wave function must be exposed to the ambient atmosphere as well as to the plethora of quantum systems of the measuring device. Under this open circumstance, the various components of the superposed wave function become affected with the elements of the environment as well as the measuring device.

This led to the initiation of a more systematic study of the effect of the environment and the measuring apparatus on quantum system resulting in the loss of quantum coherence, which is now known as *decoherence*. Use of density matrix was also initiated for decoherence by Zeh in 1970s [19,20]. Zeh continued his work on decoherence, sometimes with Erich Joos, for decades [21]. The next big step forward came when the idea of quantum entanglement was conjoined with decoherence for exploration of quantum measurement. It is fascinating to appreciate how this historic conjunction came to be recognized.

The award of the 2022 Nobel Prize in physics has brought significant attention to quantum entanglement. It is now well known that the essence of quantum entanglement arose from the famous Einstein–Podolsky–Rosen paper [22] published way back in 1935. But it remained effectively ignored by most physicists until John Bell's epochal article [23] on Bell's inequality proposed in 1964. Again it remained on the side line for quite a while due to the lack of a suitable experimental arrangement to verify Bell's proposal. Eventually a feasible experimental arrangement was devised five years later by John Clauser, Michael Horne, Abner Shimony and Richard Holt [24]. The first experimental verification of the Bell–Clauser–Horne–Shimony–Holt theory, now popularly known as quantum entanglement, was carried out by Friedman and Clauser in 1972 [25] with later substantiation by Clauser and Shimony [26]. A lack of interest of the mainstream scientists to the subject still continued perhaps because of possible loopholes in the substantiation of Bell's theorem. The checkered history of the development of this period included an underground journal to avoid the apathy of *Physical Review* editors to quantum entanglement. This is well documented in a book amusingly entitled, *How the Hippies Saved Physics* and penned by David Kaiser [27].

Eventually, a better authentication of Bell's theorem came from Alain Aspect and his group [28, 29]. Further experiments to provide loophole free confirmation of Bell's theorem led to the acceptance of quantum entanglement. An analysis of how does nature possibly accomplish nonlocal action are presented by Bhaumik [30]. The essential role of entanglement in quantum decoherence was soon realized by Kübler and Zeh [31]. However, Zeh's emphasis of entanglement in further studies of Everett's theory of quantum measurement apparently distracted him from advancing the appropriate roles of entanglement in decoherence. Nevertheless, he continued his work with others, Erich Joos being one of them [21].

4 Details of decoherence

As soon as the closed quantum system is exposed to the environment including the detector, the unitary Schrödinger evolution in a very short order generates quantum entanglement between the system and the detector including the environment making it possible to combine the system and the detector into a single bigger system. Since both the system and the detector comprising atoms and molecules abide quantum rules, one can build up a composite tensor product space using two sets of orthonormal basis vectors of Hilbert spaces. The combined system evolves in a unitary fashion. Outcome of the measurement of the selected quantum system is determined by the quantum correlations encoded in the globally entangled quantum states of the composite system. Thus, the conspicuous feature of the decoherence program is that the laws of quantum mechanics are not suspended during measurement, contrary to the popular assumption of most of the pioneers of quantum mechanics for a long time.

Since 1980, decoherence involving quantum entanglement has been extensively studied by Wojciech Zurek with his group at the Los Alamos National laboratory for almost four decades making a very substantial improvement in our understanding of the process. In summary, Zurek's investigations show that only the eigenstates or the pointer states survive in the complex environmental decoherence process and the number of entanglement states increases very substantially due to what Zurek calls quantum Darwinism. Consequently, the plethora of entanglement states consisting of the robust pointer states show up in the process of measurements. Detailed mathematical analyses of the decoherence process have been presented in a substantial number of publications by Zurek and others [32–35]. More recently, an excellent entire book on decoherence has been presented by Maximillian Schlosshauer [36]. A brief synopsis of the essential results of all these investigations will be presented next.

5 Finding the expectation values

Let us consider that the quantum system and the detector including the environment are each represented by a finite dimensional Hilbert space, $\mathcal{H}_S$ and $\mathcal{H}_E$, leading to a pure composite state $|\psi_{SE}\rangle$ that can be represented by a density matrix $\hat{\rho}_{SE}$ corresponding to the pure state as

$$\hat{\rho}_{SE} = |\psi_{SE}\rangle\langle\psi_{SE}|. \tag{1}$$

The expectation value $\langle\hat{A}\rangle$ of any observable $\hat{A}$ acting on $\mathcal{H}_S \otimes \mathcal{H}_E$ is

$$\langle\hat{A}\rangle = \mathrm{Tr}\left(\hat{\rho}_{SE}\hat{A}\right), \tag{2}$$

which is completely determined for the composite state. Despite that the composite $\hat{\rho}_{SE}$ is pure, in general, both the $\hat{\rho}_S$ and $\hat{\rho}_E$ individually are ensemble of states. Each of their reduced density matrices contains an incoherent mixture of N quantum state vectors $|\psi_{n,i}\rangle$

$$\hat{\rho}_i = \sum_{n=1}^{N} p_{n,i}|\psi_{n,i}\rangle\langle\psi_{n,i}| \tag{3}$$

where $i \in \{S, E\}$, $|\psi_{n,i}\rangle\langle\psi_{n,i}|$ are projection operators with probability $p_{n,i}$ and the sum of the probabilities is normalized, $\sum_n p_{n,i} = 1$. Thus, there can be various ensembles of states with each one having its own probability distribution that will produce the same density matrix. Therefore, for a single copy of unknown state $\hat{\rho}_{SE}$, it is the case that $\hat{\rho}_i$ are unknowable to any meaningful extent for either of the components. However, if we are given multiple copies of the same composite state $\hat{\rho}_{SE}$, then $\hat{\rho}_{SE}$ and $\hat{\rho}_i$ can be reconstructed using quantum state tomography [37] and $\langle\hat{A}\rangle$ can be obtained as the average of measurement outcomes of $\hat{A}$, where each measurement is performed on a new copy of $\hat{\rho}_{SE}$. It is rather amusing to note that we may know everything about the composite entangled pure state, while we may not know anything specific for either one of the component mixed states. Also, we may know exactly the expectation value of an observable $\langle\hat{A}\rangle$, while we may not know what the measurement outcome for each measurement run will be [38].

In the situation when the system S and the environment E are quantum correlated by entanglement, an observer having access only to the system S can compute the expectation values for any local observable using only the system's reduced density matrix

$$\hat{\rho}_S = \mathrm{Tr}_E\left(\hat{\rho}_{SE}\right) \tag{4}$$

where the reduced density matrix $\hat{\rho}_S$ is obtained by tracing out the degrees of freedom of the environment in the joint system–environment density matrix $\hat{\rho}_{SE}$. The statistics of all possible local measurements on the system S is comprehensibly encoded in the reduced density matrix. Thus, for any local observable $\hat{A}_S \otimes \hat{I}_E$ that relates only to the Hilbert space $\mathcal{H}_S$ of the quantum system S, the reduced density matrix $\hat{\rho}_S$ will be sufficient to calculate the expectation value of the observable

$$\langle\hat{A}_S\rangle = \langle\hat{A}_S \otimes \hat{I}_E\rangle = \mathrm{Tr}\left(\hat{\rho}_S\hat{A}_S\right) \tag{5}$$

Although the concept of the reduced density matrix was introduced by Paul Dirac in 1930 [39], oddly its significance does not appear to have been appreciated for almost half a century until the advent of quantum entanglement. An essential element of Zurek's milestone contributions to the decoherence program turned out to be the utilization of entanglement and consequently the reduced density matrix for dealing with expectation values among others.

6 The problem with decoherence

Although Zurek and his colleagues have advanced the decoherence program in leaps and bounds over the last four decades, there are still some conspicuous complexities in resolving the measurement problem. To begin with, although the trace rule provides a convenient way to obtain the reduced density matrix and hence the expectation value of an observable, the trace operation is a non-unitary process stroking a whiff of the collapse theory. More importantly, their work does not seem to provide a satisfactory explanation of where does the probability in measurement come from. Zurek's derivation has been criticized, among others, by Steven Weinberg. In his classic textbook *Lectures on Quantum Mechanics*, Weinberg states

> There seems to be a wide spread impression that decoherence solves all obstacles to the class of interpretations of quantum mechanics, which take seriously the dynamical assumptions of quantum mechanics as applied to everything, including measurement. [40, p. 88]

Weinberg goes on to characterize his objection by asserting that the derivation of Born's rule by Zurek is

> clearly circular, because it relies on the formula for expectation values as matrix elements of operators, which is itself derived from the Born rule. [40, p. 88]

Maximilian Schlosshauer has become a champion advocate of the application of decoherence toward the resolution of the measurement problem among others. In a paper on Zurek's derivation of the Born rule, he and Arthur Fine comment

> Certainly Zurek's approach improves our understanding of the probabilistic character of quantum theory over that sort of proposal by at least one quantum leap. [41]

However, they also criticize Zurek's derivation of the Born rule of circularity, stating

> we cannot derive probabilities from a theory that does not already contain some probabilistic concept; at some stage, we need to "put probabilities in to get probabilities out." [41]

In a recent paper [42], we have presented a plausible solution that supplements decoherence with some basic aspects of the well-established Quantum Field Theory of the Standard Model of Particle Physics. Our argument relies on some characteristics of the universal quantum fields that predetermine the values of the complex coefficients involved in the inherent superposition of eigenstates before measurement. This has been also briefly hinted by Leonard Susskind [43] by stating that the probability of a quantum state does not change during unitary evolution, which is its attribute. Thus, one of the major obstacles in using decoherence for quantum measurement could be considered resolved.

The other significant problem is that although the reduced density matrix gives a convenient way to find the expectation value of an observable, unfortunately, decoherence does not provide the pointer states separately. We only get those states still entangled with the environment including the detector states and that is not what an experimenter will measure. For that purpose, we need separable or product states such as

$$|\psi_S\rangle \otimes |\psi_E\rangle \tag{6}$$

We now present some plausible ways to accomplish this.

7 Product states using quantum rules

From the available facts so far, it appears fruitful to bring about the innate existence of the ubiquitous *vacuum quantum fluctuations* for this objective. During the unitary evolution of some superposed quantum states, no substantial effect of the fluctuations has been observed other than their essential participation in *spontaneous emission*. The vital part played by the quantum fluctuations in facilitating spontaneous emission, which is a unitary process according to quantum electrodynamics, has been known from the early days of quantum mechanics. It was conveyed in a recent article by the author [42], how some additional properties of matter like the well-known Lamb shift, anomalous g-factor, etc., would not exist without the ubiquitous fluctuations of the electro-magnetic quantum fields. These quantum fluctuations, essential for spontaneous emission, could very likely separate the pointer states from the entanglement with the environment.

The quantum fluctuations are known to be represented by a Gaussian function. The effect of the Gaussian quantum fluctuations has never been witnessed to affect the unitary Schrödinger evolution to any appreciable degree. But its significant effect could be cumulatively operative during the measurement process when a substantial number of the entangled states have been produced. Thus, it seems reasonable to explore if the quantum fluctuations could make the disentanglement effective in aiding quantum measurement in somewhat of a manner envisioned by the Ghirardi–Rimini–Weber proponents but without any modification of the Schrödinger equation as well as not requiring a very large number of quantum states during the final measurement.

In our goal to understand the effect of quantum fluctuations to produce disentanglement in recovering the product states, it seems prudent to explore some of the relevant new activities being pursued by the quantum computation community. After Peter Shor's publication of his celebrated algorithm for quantum computing in 1994, extensive studies have been carried out in both decoherence and disentanglement, which is of critical importance to quantum technology for avoiding loss of quantum coherence. Hence the activities on these topics have exploded exponentially in the last two decades. As a resource, quantum entanglement has now been measured, increased, decreased or even distilled and teleported [44].

The necessity for investigating decoherence as well as disentanglement for quantum technologies is opposite to our requirement in quantum measurement but there could be a commonality. The quantum fluctuations, so essential for spontaneous emission, could very well be involved in terminating the entanglement with the detector states. Particularly, it could be fruitful to pursue the surprising experimental observation called ESD, which stands for *early stage disentanglement* or *entanglement sudden death,* that has been observed by several groups. In these experiments, astonishingly a very swift disappearance of entanglement altogether has been reported [45–52]. Other works [53] report entanglement breaking channels.

In the simplest experimental setup, two entangled atoms in their excited states are placed one each in two widely separated cavities without any direct interaction. When the two atoms reach their ground states by spontaneous emission, surprisingly the entanglement suddenly disappears completely and the two atoms in their ground state constitute product states. Although not yet fully understood, the sudden disappearance of the entanglement is an experimental fact that could possibly be caused by a process like what occurs in the unitary quantum electrodynamical depiction of spontaneous emission. If that turns out to be true, since unitary process preserve the probability, the final reduced quantum state would have the same probability all the way from superposition to reduction. In contrast, the von Neumann collapse postulate assumes a non-unitary process following Born's rule.

The act of spontaneous emission appears to be a sudden non-unitary jump, however, if one were to keep track of all the vacuum modes, as per quantum electrodynamics the combined atom–vacuum system in fact undergoes a unitary time evolution. Thus, there could be a plausible chance that the ESD process might be unitary although the details are not yet fully understood. Further studies are planned to explore this propitious possibility.

Another feasible process resembling some aspect of the Ghirardi–Rimini–Weber procedure appears promising. This is the experimentally observed disentanglement caused by quantum fluctuations. Like a Markovian process, the quantum fluctuations does not affect the normal Schrödinger evolution indicating a limitation of the strength of the relevant interaction. It appears to take place only after enough states are made available by quantum Darwinism, when the cumulative strength of interaction would cause the disappearance of entanglement leading to the desired separable states. However, since the same information about the pointer observable is stored independently in many fragments of the environment, suitable detectors can measure the observable in different fragments even without any observer involved.

From the experimentally observed results of the consistent effect of quantum fluctuations in diminishing quantum entanglement, this approach appears to be assured for accomplishing the desired separable states. Our goal is to capture the disentangled observables in the detector. So we need to find out what could cause the disentanglement. Several experiments clearly confirm that the vacuum quantum fluctuations cause the disentanglement. Thus, the essential agent has been clearly identified and we could leave at that. But the work would be more complete if we can provide the rate and consequently the disentanglement time that could be reasonably short.

We know that the quantum fluctuations can be represented by a Gaussian. So we need to find the rates and value of the constants for the Gaussian and then possibly making some calculations like in the Ghirardi–Rimini–Weber model to predict the time taken for disentanglement. It is not essential but would complete the program. However, because studies of the details of the disentanglement process is still continuing vigorously and many of the results does not clearly identify whether it was done in a cavity where quantum electrodynamics can give more than a number of rates say for spontaneous emission. Again, the most important part is to experimentally identify the mechanism that causes disentanglement and that we already have accomplished with reasonable confidence. So the principal objective can be considered reasonably accomplished.

8 Concluding remarks

It is evident by now that the advent of quantum entanglement has led to a quantum leap for a resolution of the enduring measurement problem through the decoherence procedure. Additionally, the distinctive appeal of this program revealing that the quantum rules are not suspended during the measurement process is unique. Although many others have contributed, the decades of concerted efforts by Zurek and his group have advanced the progress of the decoherence program to a fairly mature stage.

The primary deprecation of their advancement concerns, however, is the lack of a satisfactory answer to the origin of probability and the occurrence of separable product states in the measurement process. A cogent perspective is presented here that appears to alleviate the deficiency of achieving the expected observables and their probabilities in measurement. Therefore, along with the prior article [42] by the author, a solution of the century old quantum measurement problem could be on hand. Significantly, much of the process of the reduction of the wave function or quantum to classical transition occur following quantum rules in contrast to the visions of the pioneers of quantum physics.

The universe is quantum at the core and so are we. About seven octillions of electrons and a plethora of other elementary particles inhabit our body. Our existence in the familiar classical world is made possible by continuing transition from the quantum to classical domains additionally, of course, with the irreversible metabolic processes. The quantum origin of objects in the classical arena is patently supported by the recent observation [54] of a sliver of residual quantum activity in a man size 40-kilogram mirror in the Laser Interferometer Gravitational-Wave Observatory (LIGO). In fact, in a variety of experiments, quantum effects have been observed from mesoscopic to macroscopic entities clearly indicating a transition from quantum to classical is the abiding rule when a quantum system is exposed to a huge number of quantum particles [55–57].

Most significantly, deriving the wave function of a non-relativistic quantum mechanics from the fundamental reality accessible to us so far by the standard model of particle physics and utilized by the author in a series of publications [42, 58–60], illustrate quantum mechanics could be considered weird no more. We must recognize that there are two distinct parts of reality, the quantum and the classical with their characteristic rules, but one transitioning to the other. The perception of weirdness arise when we try to understand our daily classical world through the lens of quantum rules. The quantum theory could be as splendid a theory based on fundamental realty as has been both the non-relativistic Newton's laws as well as Maxwell's theory of electrodynamics.

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